[06-09-08-T10]

Solving compound inequalities

A compound inequality is several inequalities joined by the logical operators "and" or "or". The logical operator "and" is often represented by the symbol "$\wedge$". The "or" is often represented by "$\vee$".

[EX1] Find all values of x for which the following is true. $x+3<7$ and $x+3>1$.

Solution.

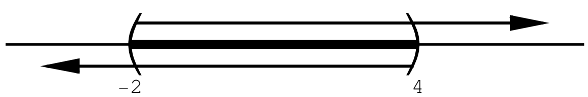
$$x+3<7 \Longrightarrow x<4 \Longrightarrow x \in (-\infty,\ 4)$$

$$x+3>1 \Longrightarrow x>-2 \Longrightarrow x \in (-2,\ \infty +)$$

$(x+3<7 \ \text{ and } \ x+3>1) \Longrightarrow x \in (-\infty,\ 4) \bigcap (-2,\ \infty +)$

$\therefore\ x \in (-2,\ 4)$

Note: a graph typically makes determining $(-\infty,\ 4) \bigcap (-2,\ \infty +)$ easy. In this example,



[EX2] Find all values of x for which the following is true. $-3<x+5<7$.

$-3<x+5<7$ means two inequalities connected by "and".

That is $-3<x+5<7 \Longleftrightarrow -3<x+5$ and $x+5<7$. As such, they can be split apart and handeled as in [EX1]. They can also be worked, as follows, without spliting them apart.

$$-3<x+5<7$$

$$\Longleftrightarrow -3-5<x<7-5$$

$$\Longleftrightarrow -8<x<2$$

$\therefore\ x \in (-8,\ 2)$

[EX3] Find all values of x for which the following is true. $4<2\,x+5<15$.

Solution.

$4 < 2x + 5 < 15$

$\Longleftrightarrow -1 < 2x < 10$

$\Longleftrightarrow -\frac{1}{2} < x < 5$

$\therefore\ x \in \left(-\frac{1}{2}, 5\right)$

- **Sometimes,as the next example shows, the compound ineqaulities must be split apart.**

[EX4] Find all values of x for which the following is true. $4x < 2x < 15$.

Trying to work them together, leads to various dead ends.

$4x < 2x < 15 \Longleftrightarrow 2x < x < \frac{15}{2}$ Bust!

$4x < 2x < 15 \Longleftrightarrow 4x - 2x < 0 < 15 - 2x$ Bust!

Solution.

Spliting apart the inequalities works well.

$4x < 2x < 15 \Longleftrightarrow (4x < 2x \wedge 2x < 15)$. Then,

$(4x < 2x \wedge 2x < 15) \Longleftrightarrow \left(x < 0 \bigwedge x < \frac{15}{2}\right)$

$\therefore\ x \in (-\infty, 0)$

[EX5] Find all values of x for which the following is true. $3x + 5 \le 7$ or $5x + 2 > 9$.

Solution.

$3x + 5 \leq 7$

$\Longleftrightarrow 3x \leq 2$

$\Longleftrightarrow x \leq \frac{2}{3}$

$5x + 2 > 9$

$\Longleftrightarrow 5x > 7$

$\Longleftrightarrow x > \frac{7}{5}$

$3x + 5 \leq 7 \bigvee 5x + 2 > 9 \Longleftrightarrow x \leq \frac{2}{3} \bigvee x > \frac{7}{5}$

$\therefore\ x \in \left(-\infty, \frac{2}{3}\right] \cup \left(\frac{7}{5}, \infty +\right)$

[06-09-08-T10-Problems]

Solving compound inequalities

- **Answer the following using interval notation.**
  A graph is <u>not</u> required, but may be helpful to you.

- **Solve the following.**

[1] $5 < 9 - 2x < 12$

[2] $-5 \le 2x - 1 < 10$

[3] $2x < 5x - 1 < 11$

[4] $2x + 1 < 7 \bigvee 5x - 2 > 9$

[5] $2x + 1 < 7 \bigvee 5x - 2 < 9$

[6] $x + 3 < 7 \bigwedge 2x - 2 < 13$

[7] $-10 < \frac{2x+1}{3} < 0$

[8] $x - 13 \le 7 \bigwedge x - 5 \ge 15$

[9] $x - 3 \le 7 \bigwedge 3x > 30$

[10] $x + 3 < 7 \bigwedge x + 3 < 5$

[06-09-08-T10-Answers]

Solving compound inequalities

[1] $\left(-\frac{3}{2}, 2\right)$

[2] $\left[-2, \frac{11}{2}\right)$

[3] $\left(\frac{1}{3}, \frac{12}{5}\right)$

[4] $\mathbb{R}$ In interval notation $(-\infty, \infty +)$

[5] $(-\infty, 3)$

[6] $(-\infty, 4)$

[7] $\left(-\frac{31}{2}, -\frac{1}{2}\right)$

[8] 20 In Interval notation [20, 20]

[9] $\emptyset$

[10] $(-\infty, 2)$